Prof. Dr. Alfred Toth

On symmetry in polycontextural semiotic matrices

1. Kaehr (2008) has given the following examples for a 3- and a 4-contextural 3-adic semiotic matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix} \begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

However, since there are neither formal nor semantic needs for the placing of the contextural environments to the sub-signs, in Toth (2009), I have shown many other types of both 3- and 4-contextural 3-adic matrices.

2. The maximal length of the contextural indices of an n-contextural matrix is (n-1), and this length is reserved for the contextural values of the main diagonal, the reason being the decomposability of the m×m-Matrix into (m-1)×(m-1), (m-2)×(m-2), etc. submatrices (cf. Kaehr 2009). So, all other $(n^2 – n)$ elements of an n-contextural matrix get contextural indices of length (n-2). In the above example, the 3-contextural matrix to the left has 2-digit-length indices in the main diagonal and 1-digit-length indices otherwise.

2.1. Thus, when we start with a 3-contextural 3-adic matrix, we get the following possibilities of 1- and 2-digit-length indices:

1; 2; 3
1,2/2,1; 1,3/3,1; 2,3/3,2,

thus 6 values (1,1; 2,2; 3,3 are excluded, because this would mean that one and the same element lies two times in the same contexture). The values of the forms (a,b) and (b,a) we have taken here together, since they are just variations of one another – namely morphisms and hetero-morphisms. When we now look at the “raw scaffolding” of a 3×3 matrix:

$$\begin{pmatrix} id1 & (a.b) & (c.d) \\ (b.a) & id2 & (e.f) \\ (d.c) & (f.e) & id3 \end{pmatrix}$$

then we see that such a matrix contains (3 · 3) – 3 = 6 different elements, i.e. elements that cannot be combined to pairs of morphisms and hetero-morphisms.

2.2. In a 4-contextural 4-adic matrix, we have the following 1-tupels:

1; 2; 3; 4,

the pairs (1,2); (1,3); (1,4); (2,3); (2,4), (3,4),

and the triples: (1,2,3); (1,2,4); (1,3,4); (2,3,4), thus 14 values.

A 4×4 Matrix has (4 · 4) – 6 = 10 different elements.

3. However, the question arises how to construct a semiotic matrix with contextuated sub-signs in the most simple and most elegant way.

3.1. In the case of the 3-contextural matrix, there are no problems, since the 6 values can be just divided over the 6 places considering that

- the trichotomy of Firstness is connected with the trichotomy of Thirdness by “the lowest interpretant (1.3)” (Bense)
- the trichotomy of Secondness builds a system of partial relations in the whole of the triadic relation because of (M ⇒ (M ⇒ O))
- the trichotomy of Thirdness is not directly connected with the partial relations of the trichotomy of Firstness, but with the trichotomy of Secondness

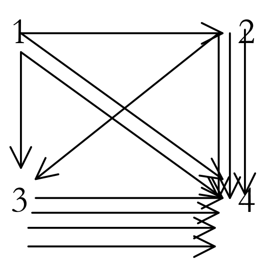
3.2. The problems start with 4-contextural matrices, since here we stand before the question of how to distribute the 14 values over 10 places. If we consider that a 4-contextural matrix can also appear as a 3-adic matrix with even less places (6), so that a 3-contextural 3-adic matrix is a fragment of a 4-contextural 4-adic matrix, we may refuse 1-tuples as contextural values. Hence we have exactly 10 values and 10 places. And as long as we are dealing with an n-contextural n-adic matrix, our more semantic argumentation may still apply here as it did above in 3.1.

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\ 3.1_{3,4} & 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \\ 4.1_{1,2} & 4.2_{2,4} & 4.3_{2,3} & 4.4_{1,2,3} \end{pmatrix}$$

3.3. However, if we have a 4-contextural 3-adic matrix and hence 6 instead of 10 free places, we must say good-bye to 4 pairs – the question is only: to which pairs? Kaehr (2008) has solved the problem in striking simplicity, by just adding 4 as another contextural value to the contextural values of the 3-contextural 3-adic matrix:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

As one sees, the pairs (1,2); (1,3); (2,3) have disappeared, and so has the triple (1,2,3). However, if we draw the 4-contextural 3-adic matrix as a graph with the vertices 1, 2, 3, 4:



then this graphs looks highly redundant, since the following graph, too, contains all morphisms necessary for the 4-contextural 3-adic matrix:

$$
\begin{tikzcd}
1 \arrow[r] \arrow[d] \arrow[dr] & 2 \arrow[dl] \arrow[d] \\
3 \arrow[r] & 4
\end{tikzcd}
$$

This graph contains the pairs (1,2); (1,3); (1,4); (2,3); (2,4); (3,4); and the triples (1,2,4); (1,2,3), (1,3,4), and (2,3,4), thus, exactly the values of the 4-contextural 4-adic matrix. Therefore, the solution just to add one contextural value to the 1-tuples (→ pairs) and pairs (→ tripels) seems not be the ideal solution in order to point out that a 4-contextural 3-adic matrix is a fragment of a 4-contextural 4-adic matrix. For such cases, Kaehr's other solution, the decomposition of a matrix in its sub-matrices, seems to be a more appropriate way. Therefore we start with

$$
\begin{pmatrix}
1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\
2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\
3.1_{3,4} & 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \\
4.1_{1,2} & 4.2_{2,4} & 4.3_{2,3} & 4.4_{1,2,3}
\end{pmatrix}
$$

and do not list all the possible 3×3-fragments of this 4×4-matrix, but just compare the red and the blue sub-matrices:

The red matrix is:

$$
\begin{pmatrix}
1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\
2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} \\
3.1_{3,4} & 3.2_{1,3} & 3.3_{2,3,4}
\end{pmatrix}
\neq
\begin{pmatrix}
1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\
2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\
3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4}
\end{pmatrix}
$$

The blue matrix is:

$$\begin{pmatrix} 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\ 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\ 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \end{pmatrix}$$

However, as it stands here, the matrix is unusable. We thus have to transport the third triad to the left, and then to apply a "normal form-operator" (cf. Toth 2004). Then, the result is:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} \\ 3.1_{3,4} & 3.2_{1,3} & 3.3_{2,3,4} \end{pmatrix} \neq \begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

but the blue and the red matrix are now the same. Therefore, in both cases we get a matrix which is not the same as Kaehr's 4-contextural 3-adic matrix.

Bibliography

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

Kaehr, Rudolf, Diamond text theory. http://www.thinkartlab.com/pkl/media/Textems/Textems.pdf (2009)

Toth, Alfred, Strukturen thematisierter Realitäten in der polykontexturalen Semiotik. In: Grundlagenstudien aus Kybernetik und Geisteswissenschaft 44/4, 2004, pp. 193-198

Toth, Alfred, (668) 2009 Inner and outer semiotic environments in the system of the trichotomic triads. In: Electronic Journal for Matheamtical Semiotics, www.mathematical-semiotics.com (2009)

8.4.2009